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$$\begin{aligned}\frac{1}{4.96} &= y - \frac{100}{96}y^2 + \frac{35}{96}y^3 - \frac{5}{96}y^4 + \frac{1}{4.96}y^5, \\ -\frac{1}{96} &= y - \frac{40}{96}y^2 - \frac{20}{96}y^3 + \frac{10}{96}y^4 - \frac{1}{96}y^5, \\ +\frac{1}{64} &= y + 0y^2 - \frac{20}{64}y^3 + 0y^4 + \frac{1}{64}y^5, \\ -\frac{1}{96} &= y + \frac{40}{96}y^2 - \frac{20}{96}y^3 - \frac{10}{96}y^4 - \frac{1}{96}y^5, \\ \frac{1}{4.96} &= y + \frac{100}{96}y^2 + \frac{35}{96}y^3 + \frac{5}{96}y^4 + \frac{1}{4.96}y^5.\end{aligned}$$

If we now develop each of these equations by (R) and add to the results the corresponding integers, 2, 4, 6, 8 and 10, we shall have the five roots of the equation. Thus,

$$\begin{aligned}x_1 &= 2 + \frac{1}{4.96} + \frac{100}{4^2 \cdot 96^3} + \frac{16640}{4^3 \cdot 96^5} + \dots = 2.002611, \\ x_2 &= 4 - \frac{1}{96} + \frac{40}{96^3} - \frac{5120}{96^5} + \dots = 3.989591, \\ x_3 &= 6 + \frac{1}{64} + \frac{20}{64^4} + \frac{1136}{64^7} + \dots = 6.015626, \\ x_4 &= 8 - \frac{1}{96} - \frac{40}{96^3} - \frac{5120}{96^5} - \dots = 7.989568, \\ x_5 &= 10 + \frac{1}{4.96} - \frac{100}{4^2 \cdot 96^3} + \frac{16640}{4^3 \cdot 96^5} - \dots = 10.002597.\end{aligned}$$

In like manner all irrational roots may be found by the rule expressed by (R) which may therefore be called the Root Theorem.

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### GEOMETRICAL DETERMINATION OF THE AREA OF THE PARABOLA.

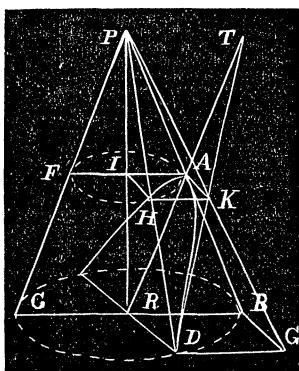
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LET  $AR$  represent the principal axis of a parabola, so placed as to form one of the equal sides of an isosceles triangle  $ABR$ , with the base  $BR$ , equal to the corresponding ordinate of the given parabola, and at  $R$  erect a perpendicular  $RP$  to intersect  $AB$  produced in  $P$ .

Suppose the cone  $PBDC$  completed by revolving  $PBR$  about  $PR$ ; then will  $CPB$ , also, be isosceles, and similar to  $RAB$ , hence the line  $AR$  is parallel to  $PC$ , and  $AB$  is half of  $PB$ .

Along the line  $AR$  pass a plane with its cutt'g



edge at right angles with  $CB$ , and the section produced,  $AED$ , is the given parabola,  $RB$  being equal to  $RD$ . Through  $A$  pass a plane parallel to the base  $CBD$  cutting the circle  $AHF$ . Since  $PIA$  and  $PRB$  are similar triangles,  $IA = \frac{1}{2}RB$ .

Suppose the conical surface to be divided into an infinite number of triangles, as  $Prs$  which is supposed to terminate at the point  $D$  on the curve, with its base parallel to  $CB$ . On  $RB$  and  $RD$  form the square  $RBGD$ , and on  $AI$  and  $IH$ , radii at right angles, form the square  $IAKH$ , and complete the prismoid  $HKGDRBIA$ . The face  $HKDG$  will be an expansion of the plane  $Prs$ , and the plane that cuts the parabola must also cut the prismoid from the corner  $K$  to the corner  $D$ , hence the line that is tangent at the point  $D$  of the parabola will pass through  $K$ . Prolong the axis  $RA$  to meet  $DK$  produced in  $T$ . Because  $AK (= AI)$  by construct'n is parallel to  $RD (= RB)$ , and because it has been shown that  $IA = \frac{1}{2}RB$ ,  $\therefore AK = \frac{1}{2}RD$ ;  $\therefore RA = \frac{1}{2}RT$ , or the subtang't is bisected at  $A$ . The same relation can be shown for the tang't of any other point of the curve by erecting a cone in like manner on its coordinates, and demonstrating as above. When the ordinate exceeds the abscissa a cone with elliptic base must be used.

Let the parabola  $ABCD$  (pr'l axis  $AR$ ) be divided into an infinite number of parts,  $BC$  being one of these parts, and  $BI$  and  $CH$  ordinates at  $B$  and  $C$ , resp'y.

From  $E$ , the middle point of  $BC$ , let fall the perpendic'lr  $EF$ , and draw the tangent  $ET$ , then is  $FT = 2FA$ . On  $DR$  and  $RA$  complete the parallelogram  $DRAL$  and draw  $CL$ ,  $Em$  and  $MBK$  parallel to  $RA$ ; then is  $Lm = mK$ .

The triangles  $CMB$  and  $EFT$  are similar, hence  $CM : MB :: EF : FT$ ;

$$\therefore CM \times FT = MB \times EF. \quad (1)$$

The trapezoid  $CBHI$  is equal to  $MB \times EF$  [by (1)]  $= CM \times FT$ . (2)

The trapezoid  $CBKL$  is equal to  $CM \times Em$ , or since  $Em = FA = \frac{1}{2}FT$ ,

$$\therefore CBKL = CM \times \frac{1}{2}FT. \quad (3)$$

Hence, from (2) and (3), the trapezoid  $CBHI = 2CBKL$ . And as the same relation holds for all similar trapezoids drawn in the parallelogram  $DRAL$ , it follows that the area of the parabola is two thirds of its circumscribed parallelogram.

